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Research Statement

Many philosophers of mathematics are occupied with questions like the following.

> What is a number?
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> What is mathematical necessity?
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> What is the infinite?

But the answers they seek are *not* the answers we would give to those questions as they might be asked outside the philosophical study. For instance, if an elementary student or a non-native speaker asked, "What is a number?", we might answer as follows:

> 1 is a number, so is 2, as well as 3, and so on [holding up fingers for each]. We use numbers to count things or put them into an order. But there are more complicated types of numbers such as rational numbers like 2/3, irrational numbers such as the square root of 2, real numbers, such as π or 3.14159…, among others.

Answers such as these could be made more precise or elaborated if needed. But they do not typically present any general problem or difficulty.

However, philosophers of mathematics would generally regard these answers as failing to get to the heart of each of the topics above. The answer about numbers, for instance, only provides a list of central examples, rather than a unifying definition which tells us what all numbers have in common by virtue of being numbers. I will call this reaction to the ordinary answers "the Socratic demand": the demand for a deeper, more strict, and more general answer to these questions than would ordinarily be provided. I call it "Socratic" because it resembles Socrates' classic demand on his interlocutors to answer the question "what is X?" in general terms (for some X, such as 'piety', 'justice', etc.), rather than attempting to explain X merely by providing a series of examples.

Note that in certain contexts there might be perfectly good reasons for strictly and generally defining some concept or other. Indeed, there are numerous topics in mathematics where such a definition is helpful for mathematical practice itself. Aside from being pedagogically useful, such definitions might allow for a variety of interesting and fruitful extensions to mathematical theory, creating new proof strategies for theorems that were not previously available. Outside mathematics, there might be powerful reasons to decide on more precise guidelines for the use of concepts such as "legal", "polite", "fair", and the like, as this might aid in our general conduct, perhaps by reducing overall harms to the general public, by increasing the likelihood of fair trials, or enabling coordinated responses to an emergency, and so on.

Could there be good reasons for strictly and generally defining "game", "chair", or "building"? Maybe so: if someone is trying to decide on the total annual value of the gaming industry, for instance, it might make sense for them to operationalize the concept of a "game" so as to produce publicly verifiable results. But notice that this operationalization of the concept will have no authority apart from that specific purpose. A very different precisification might be justified for a teacher who wants to define "games" so as to mark off acceptable activities for children to play. In neither case would we view the relevant precisification as telling us what a game "*really* is", but instead as giving us a particular precisification to satisfy particular purposes.

My research addresses the question of what reason there is, if any, for meeting the Socratic demand in the philosophy of mathematics – when this is neither a matter of practical utility nor an attempt to improve mathematical practice itself. In the case of basic mathematical concepts such as those of number, necessity, and infinity, why are the answers we would give to these questions outside the philosophical study seen as inadequate, as failing to reveal what these things *really* are? Or, even if one thinks that it is impossible to satisfy the Socratic demand, why is it thought that fulfilling the demand would give us a better understanding of these topics, if only it were possible?

I argue on the basis of considerations that I find in Wittgenstein's *Philosophical Investigations* and his *Remarks on the Foundations of Mathematics* that the Socratic demand in the philosophy of mathematics arises from the following confusions, among others: (i) tempting yet misleading analogies between different types of words or sentences (especially analogies made between mathematical and non-mathematical language), (ii) metaphorical descriptions of mathematical results that encourage perplexing yet misbegotten pictures of them, and (iii) a persistent assumption that the *objects* corresponding to our words reveal their meanings rather than the *uses* which are made of those words. Each of these three sources encourages us to think that the various subjects (number, necessity, infinity) are somehow hidden beneath the mere examples we use to illustrate them or that they admit of potentially severe puzzles which can only be resolved via stricter definitions or general descriptions of their respective essences.

For instance, analogizing a number with another 'object' such as a table, one might be encouraged to think that a number is somehow akin to a table – since they are both 'objects' – but cannot be seen, touched, or otherwise interacted with. If this is so, it is quite puzzling how we could ever know about such a thing, since (by analogy with other paradigms of 'knowledge') we typically learn about objects of the world through some kind of experience or interaction with them. This sets the stage for the Socratic demand on the question, "What is a number?". A failure to meet that demand allegedly leaves us with mystery about what numbers could possibly be or how we could ever know anything about them.

But this sense of mystery is a product of confusion. How does this confusion arise, and give rise to a sense of mystery that only a Socratic definition could dispel? To be clear, this is not a question about the psychological idiosyncrasies of particular philosophers. Rather, my question is: what it is about the practice of, say, number-talk itself that invites the confusions that elicit the sense of mystery, and the Socratic demand. Addressing this question is thus a positive inquiry and requires a kind of philosophical anthropology: an account of our linguistic practices that reveals the illusions which give rise to the Socratic demand.

Just as philosophers might investigate the nature of virtue, a philosopher can also attempt to understand what it is in our thinking about virtue that makes it reasonable to ask for a strict and general account of virtue. So the question, "What is virtue?", is not the only question a philosopher might seek to answer. They might instead ask, from a Wittgensteinian perspective, "What is it about this concept and our use of it that makes the Socratic demand appear legitimate in the first place?". Such a philosopher is looking for the source of an illusion, not from the agent who suffers the illusion – but from the illusion-inducing features of the concept as it features in our life with it. From this Wittgensteinian perspective, seeking to satisfy the Socratic demand for our concept of number is akin to asking of the lines in the famous Müller-Lyer illusion, "What makes *this* line longer than *that* one?". An alternative – and in this case more fruitful – inquiry would be to investigate the source of the illusion that one line is longer than the other.