

PHIL 5222: Philosophy of Mathematics

Fall 2025 | Tues/Thurs 2:30 – 3:45 pm | Heller 830

Instructor:

Dr. Philip Bold

Email: bold0169@umn.edu

Office: Heller 805

Office hours: Tues/Thurs 10:10 – 11:00 am

Course Description:

This course has two parts. First, we will survey the historically dominant theories in the metaphysics and epistemology of mathematics, along with the major challenges they have faced. These include Platonism, Kantianism, Logicism, Formalism, and Intuitionism. In the second part of the course, we will study Wittgenstein's later philosophy of mathematics in depth, contrasting his famously non-theoretical—and so-called therapeutic—approach with the theoretical frameworks introduced earlier.

STUDENT LEARNING OUTCOMES:

Students in the course **can identify, define, and solve problems.**

Addressing the Outcome

In this course, students will grapple with some of the most pressing and enduring issues in the philosophy of mathematics, which have challenged thinkers for thousands of years. A critical examination of various theories of the metaphysics and epistemology of mathematics will equip students to develop their own philosophical perspectives.

Assessment of Learning Related to the Outcome

Learning outcomes will be assessed through regular participation in class discussions and a series of written assignments. These assignments will require students to clearly articulate their own reasoning as well as evaluate and engage with others' perspectives on central philosophical questions.

Required Texts:

- Stewart Shapiro, *Thinking About Mathematics*, Oxford University Press
- Øystein Linnebo, *Philosophy of Mathematics*, Princeton University Press

All other readings will be made available on Canvas.

Course Requirements for Undergraduate Students

1. Weekly Discussion Posts (10 total) 10%
2. Critical Essay 1 (4-6 pages) 20%
3. Critical Essay 2 (4-6 pages) 20%
4. Participation 50%
 - a. Attendance
 - b. Class Discussion
 - c. Quality of Discussion Posts

Major Deadlines:

Discussion Posts	Wednesdays before midnight
Critical Essay 1	Sat Oct 18 by midnight
Critical Essay 2	Sat Dec 20 by midnight

Course Requirements for Graduate Students

1. Weekly Discussion Posts (10 total) 10%
2. Instructor Meeting 1 5%
3. Instructor Meeting 2 5%
4. Term Paper (10-20 pages) 30%
5. Participation 50%
 - a. Attendance
 - b. Class Discussion
 - c. Quality of Discussion Posts

Major Deadlines:

Discussion Posts	Wednesdays before midnight
Meeting 1	no later than Fri Oct 31
Meeting 2	no later than Sun Nov 30
Term Paper	Sat Dec 20 by midnight

Description of Undergraduate Requirements

1. Weekly Discussion Posts (10%): Weekly posts in which students will either respond to a prompt assigned by the instructor or raise a thoughtful question about the course material in preparation for class discussion.

Students are allowed to skip 2 of the assigned discussion posts. Since 12 discussion posts are assigned, students are required to complete 10 discussion posts throughout the semester.

2. Critical Essay 1 (20%): Students are required to write a 1200 to 1800-word essay that critically engages with one or more of the philosophical theories discussed in class.

3. Critical Essay 2 (20%): Students are required to write a 1200 to 1800-word essay that critically evaluates (and/or interprets) Wittgenstein's views about mathematics.

4. Participation (50%): Students are expected to attend all class meetings and actively participate in discussions. Participation grades are based on attendance, thoughtful contributions to discussions, and the quality of discussion posts. To succeed, students must complete assigned readings before each class. The instructor will provide participation grade estimates twice during the semester. **Electronic devices are not permitted in class unless the student has received special permission from the University and the instructor has been formally notified. Please bring a notebook and writing utensil to every class.**

Description of Graduate Requirements

- 1. Weekly Discussion Posts (10%):** (Same as undergraduate.)
- 2. Meeting 1 (5%):** Graduate students must arrange a meeting with the instructor to begin brainstorming their term paper, no later than Friday, October 31st.
- 3. Meeting 2 (5%):** Graduate students must arrange a meeting with the instructor to propose their term paper plan, no later than Sunday, November 30th.
- 4. Term Paper (30%):** A substantial paper (between 10 and 20 pages) that makes a serious academic contribution to either the history or the contemporary study of the philosophy of mathematics.
- 5. Participation (50%):** (Same as undergraduate.)

Course Schedule

* Schedule is subject to change. * Complete readings prior to class.

INTRODUCTION

Tues Sep 2 Course Syllabus

Thurs Sep 4 Øystein Linnebo, “Mathematics as a Philosophical Challenge”, pp. 2-15
(Optional: Stewart Shapiro, Ch 2., “A Potpourri of Questions ...”)

PLATO (& ARISTOTLE)

Tues Sep 9 Shapiro, Ch. 3, “Plato’s Rationalism, and Aristotle”

Thurs Sep 11 Plato, *Meno* (selections)
Plato, *Theaetetus* (selections)
(Optional: *Republic*, “The Allegory of the Cave”)
(Optional: Aristotle, *Metaphysics* (selections))

KANT (& MILL)

Tues Sep 16 Shapiro, “Kant and Mill”, pp. 73-91
(Optional: Linnebo, “Mathematics as a Philosophical Challenge”, pp. 15-19)

Thurs Sep 18 Immanuel Kant, *Prolegomena*, pp. 32-7, & 38-40
Shapiro, “Kant and Mill”, pp. 91-102
(Optional: Linnebo, Ch. 6, “Empiricism about Mathematics”, pp. 88-92)
(Optional: Kant, *Prolegomena*, pp. 40-45)

FREGE'S LOGICISM & RUSSELL'S PARADOX

Tues Sep 23 Shapiro, Ch. 5, "Logicism", pp. 107-115
Bertrand Russell, Letter to Frege, 16 June 1902
(Optional: Linnebo, "Frege's Logicism")
(Optional: Shapiro, Ch. 5, "Logicism", pp. 115-124)

Thurs Sep 25 Gottlob Frege, Foreword to *Basic Laws of Arithmetic*
(Optional: Frege, Introduction to *Foundations of Arithmetic*)

FORMALISM

Tues Sep 30 Shapiro, Ch. 6, "Formalism", pp. 140-148, & 158-165
(Optional: Linnebo, Ch. 4, "Hilbert's Program")

Thurs Oct 2 David Hilbert, "On the Infinite"
Shapiro, Ch. 6, "Formalism", pp. 165-168
(Optional: A.W. Moore, *The Infinite*, Ch. 8.2, 8.3, & 9.2)

INTUITIONISM

Tues Oct 7 Shapiro, Ch. 7, "Intuitionism", pp. 172-185
(Optional: Linnebo, Ch. 5, "Intuitionism")

Thurs Oct 9 L.E.J. Brouwer, "Mathematics, Science, and Language" (1928 Lecture)

MATHEMATICAL TRUTH & ONTOLOGY

Tues Oct 14 Shapiro, Ch. 8, "Numbers Exist"
(Optional: Linnebo, Ch. 6, "Empiricism about Mathematics", pp. 92-99)

Thurs Oct 16 *No Class*

FIRST PAPER DUE SATURDAY OCT 18 BY MIDNIGHT

Tues Oct 21 Shapiro, Ch. 9, "No They Don't"
(Optional: Linnebo, Ch. 7, "Nominalism")

Thurs Oct 23 Paul Benacerraf, "Mathematical Truth"
Hartry Field, "Knowledge of Mathematical Entities"

WITTGENSTEIN'S LATER PHILOSOPHY OF MATHEMATICS

Tues Oct 28 Philip Bold, "Wittgenstein's Therapeutic Conception of Philosophy"
(Optional: Ray Monk, "The Reception of Wittgenstein's Philosophy of Math")

Thurs Oct 30 Ludwig Wittgenstein, *Philosophical Investigations* (selections)

Tues Nov 4 Wittgenstein, *Remarks on the Foundations of Mathematics* (RFM), Part I

Thurs Nov 6 Wittgenstein, RFM, Part I

WITTGENSTEIN ON MATHEMATICAL NECESSITY

Tues Nov 11 Juliet Floyd, “Wittgenstein on 2, 2, 2, ...”,
Wittgenstein, *Philosophical Investigations* (selections)
Wittgenstein, RFM Part I

Thurs Nov 13 Philip Bold, “Crossing Pictures of Determination...”

WITTGENSTEIN & NATURALISM

Tues Nov 18 Penelope Maddy, *The Logical Must*, Preface, Intro, Ch. 5 & 6

Thurs Nov 20 *The Logical Must*, Ch. 6 & 7

THANKSGIVING BREAK

Tues Nov 25 *No Class*

Thurs Nov 27 *No class*

WITTGENSTEIN ON TRUTH & REALISM

Tues Dec 2 Wittgenstein, *Lectures on the Foundations of Mathematics*, Lectures I & XXV
(Optional: G.H. Hardy, “Mathematical Proof”)

Thurs Dec 4 Philip Bold, “Wittgenstein on Truth and Realism in Mathematics”
(Optional: Bold, “Truth in Wittgenstein’s *Philosophical Investigations*”)

WITTGENSTEIN ON THE INFINITE

Tues Dec 9 A.W. Moore, *The Infinite*, Ch. 8.2, 8.3, & 9.3
Wittgenstein, RFM Part I, Appendix III (selections)
(Optional: Wittgenstein, *Zettel* (selections))

Thurs Dec 11 Philip Bold, “From Pictures to Employments: Wittgenstein on the Infinite”
(Optional: A.W. Moore, “Wittgenstein’s Later Philosophy of Mathematics”)

FINAL REFLECTIONS

Tues Dec 16 Open Discussion

Thurs Dec 18 Optional Office Hour

FINAL PAPER DUE SATURDAY DEC 20 BY MIDNIGHT

Course Policies

1. **No Technology in Class:** Students may not use electronic devices during class unless they have received special permission from the University and the instructor has been formally notified.
2. **Late Work:** Late work will be penalized unless permission has been granted from the instructor.
3. **Grading:** Final grades will be calculated as follows.

Letter Grades are translated to the following numerical scores.

A+ = 100	C+ = 78
A = 95	C = 75
A- = 92	C- = 72
B+ = 88	D+ = 68
B = 85	D = 65
B- = 82	F = 50

Final letter grades will be calculated using the following scale.

97-100 = A+	77-79 = C+
93-96 = A	73-76 = C
90-92 = A-	70-72 = C-
87-89 = B+	67-69 = D+
83-86 = B	60-66 = D
80-82 = B-	50 = F

Below is a list of general criteria that will be used to evaluate your writing assignments.

- Quality of ideas (50%)
 - *Range and depth of argument; fair representation of philosopher's position; logic of argument; quality and/or originality of thought; appropriate sense of the complexity of the topic; appropriate awareness of opposing views.*
- Organization and Argumentation (40%)
 - *Clarity of thesis statement in the introduction; logical and clear arrangement of ideas; effective use of transitions; unity and coherence of paragraphs; good development of arguments through supporting details and evidence.*

- Clarity, style, and grammar (10%)
 - *Ease of readability; appropriate voice, tone, and style for the assignment; clarity of sentence structure; grammatically correct sentences; accurate spelling; careful proofreading.*
4. **Plagiarism:** Plagiarism will result in a final grade of F for the course. If you have any questions about plagiarism or how to properly cite sources, do not hesitate to ask. According to the UMN Student Conduct Code: “Plagiarism shall mean representing the words, creative work, or ideas of another person as one’s own without providing proper documentation of source.”
 5. **Use of AI:** Students are strongly discouraged from using AI platforms to complete any writing assignments.
 6. **Disability Accommodations:** If you have, or think you have, a disability that would affect your performance in this course, please contact the Disability Resource Center (DRC) <https://ars.unc.edu/> to arrange confidential discussion regarding equitable access and reasonable accommodations.
 7. **Sexual Misconduct.** Sexual misconduct (sexual harassment, sexual assault, stalking, and relationship violence) includes unwelcome sexual advances, requests for sexual favors, and/or other verbal or physical conduct of a sexual nature. Such behavior is prohibited in the University setting.

Per the [U of M policy on sexual harassment](#), I am required to share information that I learn about possible sexual misconduct with the campus Title IX office. Within the requirements of my job, I will be as responsive as possible to your requests for confidentiality and support. You can also, or alternately, choose to talk with a confidential resource that will not share information that they learn about sexual misconduct. Confidential resources include:

- The Aurora Center 24hr Helpline (612-626-9111)
- Boynton Mental Health (612-624-1444)
- Student Counseling Services (612-624-3323)

8. Other Useful Resources:

- Technology resources for remote learning: <https://it.umn.edu/working-learning-campus/get-internet-access-campus>
- University Policies: <https://policy.umn.edu/>
- Student Writing Support: <http://writing.umn.edu/sws/>
- Disability Resource Center: <https://disability.umn.edu/>
- Student Counseling Center: <https://counseling.umn.edu/>
- The Stanford Encyclopedia of Philosophy: <https://plato.stanford.edu>